NAG Toolbox for MATLAB

g03cc

1 Purpose

g03cc computes factor score coefficients from the result of fitting a factor analysis model by maximum likelihood as performed by g03ca.

2 Syntax

```
[fs, ifail] = g03cc(method, rotate, fl, psi, e, r, 'nvar', nvar, 'nfac', nfac)
```

3 Description

A factor analysis model aims to account for the covariances among p variables, observed on n individuals, in terms of a smaller number, k, of unobserved variables or factors. The values of the factors for an individual are known as factor scores. g03ca fits the factor analysis model by maximum likelihood and returns the estimated factor loading matrix, Λ, and the diagonal matrix of variances of the unique components, Ψ. To obtain estimates of the factors, a p by k matrix of factor score coefficients, Φ, is formed. The estimated vector of factor scores, $\hat{f}$, is then given by:

$$\hat{f} = x^{\mathrm{T}}\Phi,$$

where x is the vector of observed variables for an individual.

There are two commonly used methods of obtaining factor score coefficients.

The regression method:

$$\Phi = \Psi^{-1}\Lambda\left(I + \Lambda^{\mathrm{T}}\Psi^{-1}\Lambda\right)^{-1},$$

and Bartlett's method:

$$\Phi = \Psi^{-1}\Lambda\left(\Lambda^{\mathrm{T}}\Psi^{-1}\Lambda\right)^{-1}.$$

See Lawley and Maxwell 1971 for details of both methods. In the regression method as given above, it is assumed that the factors are not correlated and have unit variance; this is true for models fitted by g03ca. Further, for models fitted by g03ca,

$$\Lambda^{\mathrm{T}}\Psi^{-1}\Lambda = \Theta - I,$$

where Θ is the diagonal matrix of eigenvalues of the matrix S^{*}, as described in g03ca.

The factors may be orthogonally rotated using an orthogonal rotation matrix, R, as computed by g03ba. The factor scores for the rotated matrix are then given by ΛR.

4 References

Lawley D N and Maxwell A E 1971 *Factor Analysis as a Statistical Method* (2nd Edition) Butterworths

5 Parameters

5.1 Compulsory Input Parameters

1: **method – string**

Indicates which method is to be used to compute the factor score coefficients.

method = 'R'

The regression method is used.

method = 'B'

Bartlett's method is used.

Constraint: **method** = 'B' or 'R'.

2: **rotate – string**

Indicates whether a rotation is to be applied.

rotate = 'R'

A rotation will be applied to the coefficients and the rotation matrix, R, must be given in **r**.

rotate = 'U'

No rotation is applied.

Constraint: **rotate** = 'R' or 'U'.

3: **fl**(**ldfl**,**nfac**) **– double array**

ldfl, the first dimension of the array, must be at least **nvar**.

Λ, the matrix of unrotated factor loadings as returned by g03ca.

4: **psi**(**nvar**) **– double array**

The diagonal elements of Ψ, as returned by g03ca.

Constraint: $\mathbf{psi}(i) > 0.0$, for $i = 1, 2, \ldots, p$.

5: **e**(**nvar**) **– double array**

The eigenvalues of the matrix S^*, as returned by g03ca.

Constraint: $\mathbf{e}(i) > 1.0$, for $i = 1, 2, \ldots, p$.

6: **r**(**ldr**,$*$) **– double array**

The first dimension, **ldr**, of the array **r** must satisfy

if **rotate** = 'R', **ldr** $\geq$ **nfac**;
1 otherwise.

The second dimension of the array must be at least 1 if **rotate** = 'U' and at least **nfac** if **rotate** = 'R'

If **rotate** = 'R', **r** must contain the orthogonal rotation matrix, R, as returned by g03ba.

If **rotate** = 'U', **r** need not be set.

5.2 Optional Input Parameters

1: **nvar – int32 scalar**

Default: The dimension of the arrays **psi**, **e**. (An error is raised if these dimensions are not equal.)

p, the number of observed variables in the factor analysis.

Constraint: **nvar** $\geq$ **nfac**.

2: **nfac – int32 scalar**

Default: The dimension of the arrays **fl**, **fs**. (An error is raised if these dimensions are not equal.)

k, the number of factors in the factor analysis.

Constraint: **nfac** ≥ 1.

5.3 Input Parameters Omitted from the MATLAB Interface

ldfl, ldr, ldfs, wk

5.4 Output Parameters

1: **fs(ldfs,nfac) – double array**

The matrix of factor score coefficients, Φ. **fs**(i,j) contains the factor score coefficient for the jth factor and the ith observed variable, for $i = 1,2,\ldots,p$ and $j = 1,2,\ldots,k$.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, **nfac** < 1,
or **nvar** $<$ **nfac**,
or **ldfl** $<$ **nvar**,
or **ldfs** $<$ **nvar**,
or **rotate** $=$ 'R' and **ldr** $<$ **nfac**,
or **method** $\neq$ 'R' or 'B',
or **rotate** $\neq$ 'R' or 'U'.

ifail $= 2$

On entry, a value of **psi** ≤ 0.0,
or a value of **e** ≤ 1.0.

7 Accuracy

Accuracy will depend on the accuracy requested when computing the estimated factor loadings using g03ca.

8 Further Comments

If principal component analysis is required the function g03aa computes the principal component scores directly. Hence, the factor score coefficients are not needed.

9 Example

```
method = 'R';
rotate = 'U';
fl = [0.5533188537924767, -0.4285636478848139;
     0.568155945618757, -0.2883165515607369;
     0.3921781680583233, -0.4499647969874735;
     0.7404217621764928, 0.2728004512944507;
     0.7238701315235972, 0.2113108264190836;
     0.5953586397783871, 0.1316911981935982];
psi = [0.5101714468054336;
     0.5940723866877339;
```

```
     0.6437279642693183;
     0.3773554593674197;
     0.4313596544519335;
     0.6282055268211231];
e = [5.61418330106997;
     2.142797556801369;
     1.092257889179389;
     1.026411127333595;
     0.9908235560038364;
     0.8905078592462671];
r = [0.001066610312409233];
[fs, ifail] = g03cc(method, rotate, fl, psi, e, r)
```

```
fs =
    0.1932   -0.3920
    0.1703   -0.2265
    0.1085   -0.3262
    0.3495    0.3374
    0.2989    0.2286
    0.1688    0.0978
ifail =
           0
```